JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON THURSDAY 22nd JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 5k$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + \lambda k, \lambda \in \mathbb{R}$. Let the projection of the vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ on the diagonal $\overrightarrow{AC}$ of the parallelogram ABCD be of length one unit. If α, β, where $\alpha > \beta$, be the roots of the equation $\lambda^2 x^2 - 6\lambda x + 5 = 0$, then $2\alpha - \beta$ is equal to

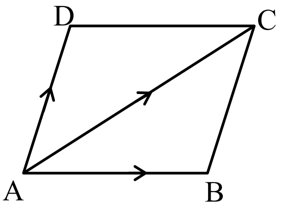
(1) 1 (2) 4

(3) 3 (4) 6

Ans. **(3)**

Sol. $\overrightarrow{AC} = 3\hat{i} + 6\hat{j} + (\lambda - 5)k$

$\vec{v} \cdot AC = 1 \Rightarrow 3 + 6 + \lambda - 5 = \sqrt{9 + 36 + (\lambda - 5)^2}$



$\Rightarrow \lambda^2 + 8\lambda + 16 = \lambda^2 - 10\lambda + 70$

$\Rightarrow \lambda = \dfrac{54}{18} = 3$

$\therefore$ Quadratic : $9x^2 - 18x + 5 = 0 \Rightarrow x = \dfrac{1}{3}, \dfrac{5}{3}$

$\therefore 2\alpha - \beta = \dfrac{10 - 1}{3} = 3$

2. Let the relation R on the set M = {1, 2, 3,.......16} be given by

R = {(x,y) : 4y = 5x – 3, x, y ∈ M}.

Then the minimum number of elements required to be added in R, in order to make the relation symmetric, is equal to

(1) 1 (2) 2

(3) 4 (4) 3

Ans. **(2)**

Sol. R = {(3, 3), (7, 8), (11, 13)}

to make it symmetric (8, 7), (13, 11) must be added.

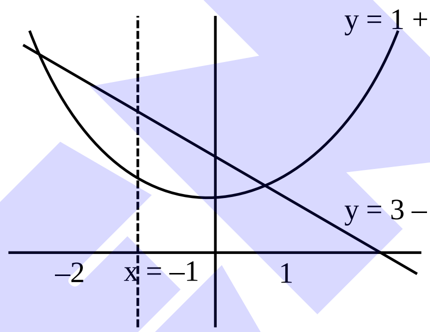
3. Let the line x = –1 divide the area of the region $\{(x, y) : 1 + x^2 \le y \le 3 - x\}$ in the ratio m : n, gcd (m, n) = 1. Then m + n is equal to

(1) 25 (2) 28

(3) 26 (4) 27

Ans. **(4)**

Sol.



$$\frac{m}{n} = \frac{\int_{-1}^{1}\left[(3 - x) - (1 + x^2)\right]dx}{\int_{-2}^{1}\left[(3 - x) - (1 + x^2)\right]dx} = \frac{20}{7}$$

$\therefore$ m + n = 20 + 7

= 27

4. Two distinct numbers a and b are selected at random from 1, 2, 3,......, 50. The probability, that their product ab is divisible by 3, is

(1) $\dfrac{561}{1225}$ (2) $\dfrac{664}{1225}$

(3) $\dfrac{272}{1225}$ (4) $\dfrac{8}{25}$

Ans. **(2)**

Sol. Req. probability = 1– (product not divisible by 3)

Multiple of 3 = 16

Not multiple of 3 = 34

$$= 1 - \frac{^{34}C_2}{^{50}C_2} = \frac{664}{1225}$$

5. Let $f(x) = x^{2025} - x^{2000}$, $x \in [0, 1]$ and the minimum value of the function $f(x)$ in the interval $[0, 1]$ be $(80)^{80}\,(n)^{-81}$. Then n is equal to

(1) –81 (2) –40

(3) –41 (4) –80

Ans. **(1)**

Sol. $f(x) = x^{2025} - x^{2000}$

$$f'(x) = 0 \Rightarrow x = \left(\frac{2000}{2025}\right)^{1/25} = \alpha(\text{say})$$

$$\therefore f(0) = 0,\ f(1) = 0,\ f(\alpha) = \left(\frac{80}{81}\right)^{80} \cdot \frac{-1}{81} = 80^{80}.(-81)^{-81}$$

6. Let $P(\alpha, \beta, \gamma)$ be the point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point $(1, -1, 0)$ and nearer to the origin. Then the shortest distance, between the lines $\frac{x-\alpha}{1} = \frac{y-\beta}{2} = \frac{z-\gamma}{3}$ and $\frac{x+5}{2} = \frac{y-10}{1} = \frac{z-3}{1}$, is equal to

(1) $7\sqrt{\frac{5}{4}}$ (2) $4\sqrt{\frac{7}{5}}$

(3) $4\sqrt{\frac{5}{7}}$ (4) $2\sqrt{\frac{7}{4}}$

Ans. **(2)**

Sol. Let $P(2\lambda + 1, -3\lambda - 1, \lambda)$

Then $4\lambda^2 + 9\lambda^2 + \lambda^2 = 16 \cdot 14 \Rightarrow \lambda = \pm 4 \Rightarrow -4$ (nearer to origin)

$\therefore$ P(–7, 11, –4)

$$\therefore \text{ Shortest distance} = \frac{\begin{vmatrix} 2 & -1 & 7 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}\right|}$$

$$= \frac{28}{\sqrt{1+25+9}} = \frac{4\sqrt{7}}{\sqrt{5}}$$

7. If a random variable x has the probability distribution

x	0	1	2	3	4	5	6	7
p(x)	0	2k	k	3k	$2k^2$	2k	k^2+k	$7k^2$

then $P(3 < x \le 6)$ is equal to

(1) 0.34 (2) 0.22

(3) 0.64 (4) 0.33

Ans. **(4)**

Sol. $\sum P(x_i) = 1$

$\Rightarrow 9k + 10k^2 = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow k = \frac{1}{10}$

$P(3 < x \le 6) = 3k + 3k^2$

$= \frac{3}{10} + \frac{3}{100} = 0.33$

$= 0.33$

8. The number of distinct real solutions of the equation $x|x + 4| + 3|x + 2| + 10 = 0$ is

(1) 3 (2) 1

(3) 0 (4) 2

Ans. **(2)**

Sol. Case I $x < -4$

$x(-(x+4)) + 3(-(x+2)) + 10 = 0$

$x^2 + 7x - 4 = 0$

$\Rightarrow x = -\frac{7+\sqrt{65}}{2}$ or $-\frac{7-\sqrt{65}}{2}$

Reject Acept

Case II $-4 \le x < -2$

$x(x + 4) + 3(-(x+2)) + 10 = 0$

$x^2 + x + 4 = 0$

$D < 0$ No solution

Case III $x \ge -1$

$x(x + 4) + 3(x + 2) + 10 = 0$

$x^2 + 7x + 16 = 0$

$D < 0$ No solution

$\Rightarrow$ No. of solution = 1.

9. Let $f : [1, \infty) \to \mathbb{R}$ be a differentiable function, If $6\int_1^x f(t)dt = 3xf(x) + x^3 - 4$ for all $x \geq 1$, then the value of $f(2) - f(3)$ is

(1) –4 (2) –3

(3) 4 (4) 3

Ans. (4)

Sol. $6\int_1^x f(t)dt = 3x\, f(x) + x^3 - 4$

Diff. both side

$6\, f(x) = 3x\, f'(x) + 3f(x) + 3x^2$

$3f(x) = 3x\, f'(x) + 3x^2$

$x\dfrac{dy}{dx} - y = -x^2$

$\dfrac{x\dfrac{dy}{dx} - 9}{x^2} = -1$

$\Rightarrow \dfrac{d}{dx}\left(\dfrac{y}{x}\right) = -1$

$\dfrac{y}{x} = -x + C$

$\Rightarrow f(x) = -x^2 + Cx$

at $x = 1$, $y = 1 \Rightarrow C = 2$

$f(x) = -x^2 + 2x$

$f(2) - f(3) = 3$

10. If the line $\alpha x + 2y = 1$, where $\alpha \in \mathbb{R}$, does not meet the hyperbola $x^2 - 9y^2 = 9$, then a possible value of α is :

(1) 0.6 (2) 0.8

(3) 0.5 (4) 0.7

Ans. (2)

Sol. $y = \dfrac{1 - \alpha x}{2}$

Put this in equation of hyperbola

$\therefore x^2 - 9\left(\dfrac{1-\alpha x}{2}\right)^2 = 9$

$(4-9\alpha^2)x^2 + 18\alpha x - 45 = 0$

$\because$ line does not intersect hyperbola

$\therefore D < 0$

$\Rightarrow \alpha^2 - \dfrac{5}{9} > 0$

$\Rightarrow \alpha \in \left(-\infty, -\dfrac{\sqrt{5}}{3}\right) \cup \left(\dfrac{\sqrt{5}}{3}, \infty\right)$

Here $\dfrac{\sqrt{5}}{3} \simeq 0.74$

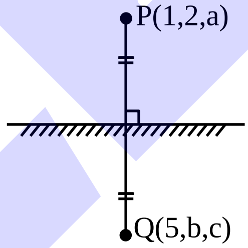
11. If the image of the point P (1, 2 , a) in the line $\dfrac{x-6}{3} = \dfrac{y-7}{2} = \dfrac{7-z}{2}$ is Q(5, b, c), then $a^2 + b^2 + c^2$ is equal to

(1) 293 (2) 264

(3) 298 (4) 283

Ans. (3)

Sol.



$\dfrac{x-6}{3} = \dfrac{y-7}{2} = \dfrac{z-7}{-2}$

Point $M \equiv \left(3, \dfrac{b}{2} + 1, \dfrac{c+a}{2}\right)$ satisfies the line

$\dfrac{3-6}{3} = \dfrac{\dfrac{b}{2} + 1 - 7}{2} = \dfrac{\dfrac{c+a}{2} - 7}{-2}$

$-1 = \dfrac{b-12}{4} = \dfrac{c+a-14}{-4}$

$\Rightarrow b = 8 \quad \ldots (1) \quad \& \quad c + a = 18 \ldots (2)$

Now $PQ \perp L$

$\Rightarrow (4i + (b-2)j + (c-a)k) \cdot (3i + 2j - 2k) = 0$

$\Rightarrow 12 + 2(b-2) - 2(c-a) = 0$

$\Rightarrow 6 + (b-2) - (c-a) = 0$

$\Rightarrow b - c + a + 4 = 0$

$\Rightarrow 8 - c + a + 4 = 0$

$\Rightarrow c + a = 12 \quad \ldots (3)$

From (2) & (3)

$c = 15$ & $a = 3$

So $a^2 + b^2 + c^2 = 9 + 64 + 225 = 298$

12. Let the set of all values of r, for which the circles $(x+1)^2+(y+4)^2=r^2$ and $x^2+y^2-4x-2y-4=0$ intersect at two distinct points be the interval (α, β). Then $\alpha\beta$ is equal to

(1) 25 (2) 20

(3) 21 (4) 24

Ans. **(1)**

Sol. $(x-2)^2+(y-1)^2=3^2$ & $(x+1)^2+(y+4)^2=r^2$

$|r_1-r_2|<c_1c_2<r_1+r_2$

$|r-3|<\sqrt{(2+1)^2+(1+4)^2}<r+3$

$|r-3|<\sqrt{34}$ & $r+3>\sqrt{34}$

$-\sqrt{34}<r-3<\sqrt{34}$ & $r>\sqrt{34}-3$

i.e. $r=\left(3-\sqrt{34},3+\sqrt{34}\right)\cap\left(\sqrt{34}-3,\infty\right)$

i.e. $r\in\left(\sqrt{34}-3,\sqrt{34}+3\right)$

$\therefore \alpha\beta=\left(\sqrt{34}-3\right)\left(\sqrt{34}+3\right)$

$=34-9$

$=25$

13. If $A=\begin{bmatrix}2&3\\3&5\end{bmatrix}$, then the determinant of the matrix $(A^{2025}-3A^{2024}+A^{2023})$ is

(1) 28 (2) 12

(3) 24 (4) 16

Ans. **(4)**

Sol. $A=\begin{bmatrix}2&3\\3&5\end{bmatrix}\Rightarrow A^2=\begin{bmatrix}13&21\\21&34\end{bmatrix}$

$|A^{2025}-3A^{2024}+A^{2023}|$

$=\left|A^{2023}\left(A^2-3A+I\right)\right|$

$=|A|^{2023}\left|A^2-3A+I\right|$

$=1.\begin{vmatrix}8&12\\12&20\end{vmatrix}=160-144=16$

14. If the domain of the function $f(x)=\sin^{-1}\left(\frac{5-x}{3+2x}\right)+\frac{1}{\log_e(10-x)}$ is $(-\infty,\alpha]\cup[\beta,\gamma)-\{\delta\}$, then $6(\alpha+\beta+\gamma+\delta)$ is equal to

(1) 70 (2) 66

(3) 67 (4) 68

Ans. **(1)**

Sol. $-1\le\frac{5-x}{2x+3}\le$ & $10-x>0$, $10-x\ne 1$

$\left|\frac{5-x}{2x+3}\right|\ge 1$ & $x<10$ & $x\ne 9$

$(5-x)^2-(2x+3)^2\le 0$ & $x<10$ & $4x\ne 9$

$(x+8)(3x-2)\ge 0$ & $x<10$ & $x\ne 9$

$\Rightarrow(-\infty,-8]\cup\left[\frac{2}{3},10\right)-\{9\}$

$\Rightarrow(\alpha+\beta+\gamma+\delta)=6\left(-8+\frac{2}{3}+10+9\right)$

$=70$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1}{[x]+4}\right)dx$, where $[\bullet]$ denotes the greatest integer function, is

(1) $\frac{1}{60}(21\pi-1)$ (2) $\frac{1}{60}(\pi-7)$

(3) $\frac{7}{60}(3\pi-1)$ (4) $\frac{7}{60}(\pi-3)$

Ans. **(3)**

Sol. $I=\int_{-\pi/2}^{\pi/2}\frac{1}{[x]+4}dx$

$I=\int_{-\pi/2}^{-1}\frac{dx}{2}+\int_{-1}^{0}\frac{dx}{3}+\int_{1}^{0}\frac{dx}{4}+\int_{1}^{\pi/2}\frac{dx}{5}$

$=\frac{1}{2}\left(-1+\frac{\pi}{2}\right)+\frac{1}{3}(1)+\frac{1}{4}(1)+\left(\frac{\pi}{2}-1\right)\frac{1}{5}$

$=\frac{7\pi}{20}-\frac{7}{60}=\frac{7}{60}(3\pi-1)$

16. The coefficient of x^{48} in $(1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + ... + 100(1 + x)^{100}$ is equal to :

(1) $100.\ {}^{100}C_{49} - {}^{100}C_{50}$ (2) ${}^{100}C_{50} + {}^{101}C_{49}$

(3) $100.\ {}^{100}C_{49} - {}^{100}C_{48}$ (4) $100.\ {}^{101}C_{49} - {}^{100}C_{50}$

Ans. **(4)**

Sol. Let $1 + x = r$

$\therefore S = 1.r + 2.r^2 + 3.r^3 + \ldots\ldots + 100r^{100}$ .... (1)

(AGP)

$rS = 1.r^2 + 2.r^3 + \ldots\ldots + 99r^{100} + 100r^{101}$ .... (2)

(1) – (2) gives

$$S = -\frac{(1+x)^{101}}{x^2} + \frac{1}{x^2} + \frac{100(1+x)^{101}}{x}$$

$\therefore$ coefficient x^{48} in S

$= -$ coefficient of x^{48} in $\dfrac{(1+x)^{101}}{x^2} + 100$.

Coefficient of x^{48} in $\dfrac{(1+x)^{101}}{x}$

$= 100\,{}^{101}C_{49} - {}^{101}C_{50}$

17. If the chord joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the parabola $y^2 = 12x$ subtends a right angle at the vertex of the parabola, then $x_1x_2 - y_1y_2$ is equal to

(1) 288 (2) 280

(3) 284 (4) 292

Ans. **(1)**

Sol. $(x_1y_1) = (3t_1^2, 6t_1)$ & $(x_2y_2) = (3t_2^2, 6t_2)$

$t_1t_2 = -4$

$x_1x_2 = 9(t_1t_2)^2,\ y_1y_2 = 36t_1t_2$

$x_1x_2 - y_1y_2 = 9(16) - 36(-4)$

$= 144 + 144$

$= 288$

18. The number of solutions of $\tan^{-1}4x + \tan^{-1}6x = \dfrac{\pi}{6}$,

where $-\dfrac{1}{2\sqrt{6}} < x < \dfrac{1}{2\sqrt{6}}$ is equal to

(1) 3 (2) 0

(3) 1 (4) 2

Ans. **(3)**

Sol. $\tan^{-1}4x + \tan^{-1}6x = \dfrac{\pi}{6}$

$$\Rightarrow \tan^{-1}\left(\frac{4x+6x}{1+24x^2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{10x}{1-24x^2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 24x^2 + 10\sqrt{3}x - 1 = 0$$

$$x = \frac{-10\sqrt{3} \pm \sqrt{300+96}}{48}$$

$$x = \frac{\sqrt{396} - 10\sqrt{3}}{48}$$

Only 1 solution in $\left(-\dfrac{1}{2\sqrt{6}}, \dfrac{1}{2\sqrt{6}}\right)$

19. Let the solution curve of the differential equation $xdy - ydx = \sqrt{x^2+y^2}\,dx$, $x > 0$, $y(1) = 0$, be $y = y(x)$. Then $y(3)$ is equal to

(1) 4 (2) 6

(3) 1 (4) 2

Ans. **(1)**

Sol. $\dfrac{xdy - ydx}{x^2} = \dfrac{\sqrt{x^2+y^2}}{x^2}dx$

$$d\left(\frac{y}{x}\right) = \sqrt{1+\frac{y^2}{x^2}}\cdot\frac{1}{x}dx$$

$$\int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \int \frac{1}{x}dx$$

$$\Rightarrow \ln\left(\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}\right) = \ln x + \ln k = \ln kx$$

$$\Rightarrow y + \sqrt{x^2+y^2} = kx^2$$

$0 + 1 = k$

$$\Rightarrow y + \sqrt{x^2+y^2} = x^2$$

$$y + \sqrt{9+y^2} = 9$$

$y = 4$

20. If the sum of the first four terms of an A.P. is 6 and the sum of its first six terms is 4, then the sum of its first twelve terms is

(1) –20 (2) –24

(3) –26 (4) –22

Ans. **(4)**

Sol. Sum of first 4 term $S_4 = 6$

$$\frac{4}{2}(2a+3d)=6 \Rightarrow 2a+3d=3 \quad \text{..... (1)}$$

Sum of first 6 terms $S_6 = 4$

$$\frac{6}{2}(2a+5d)=4 \Rightarrow 2a+5d=\frac{4}{3} \quad \text{..... (2)}$$

eq. (2) – eq. (1)

$$(2a+5d)-(2a+3d)=\frac{4}{3}-3$$

$$\Rightarrow d=-\frac{5}{6}$$

$$\therefore\ 2a+3\left(-\frac{5}{6}\right)=3 \Rightarrow a=\frac{11}{4}$$

$$S_{12}=\frac{12}{2}\left\{2\times\frac{11}{4}+(12-1)\left(-\frac{5}{6}\right)\right\}$$

$$S_{12}=6\left(-\frac{22}{6}\right)=-22$$

SECTION-B

21. Let $\alpha=\frac{-1+i\sqrt{3}}{2}$ and $\beta=\frac{-1-i\sqrt{3}}{2}$, $i=\sqrt{-1}$. If $(7-7\alpha+9\beta)^{20}+(9+7\alpha-7\beta)^{20}+(-7+9\alpha+7\beta)^{20}+(14+7\alpha+7\beta)^{20}=m^{10}$, then m is ________.

Ans. **(49)**

Sol. $$(9+7\omega-7\omega^2)+\omega^{20}(9+7\omega-7\omega^2)^{20}+$$

$$\omega^{40}(9+7\omega-7\omega^2)^{20}+(14+7(\omega+\omega^2))^{20}$$

$$(9+7\omega-7\omega^2)^{20}(1+\omega+\omega^2)+(14-7)^{20}=7^{20}$$

$$=(49)^{10}$$

Hence, M = 49

22. Let A be a 3 × 3 matrix such that $A + A^T = O$. If $A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}3\\3\\2\end{bmatrix}, A^2\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}-3\\19\\-24\end{bmatrix}$ and $\det(\text{adj}(2\text{adj}(A + I))) = (2)^{\alpha}.(3)^{\beta}.(11)^{\gamma}$, α, β, γ are non-negative integers, then $\alpha + \beta + \gamma$ is equal to ______

Ans. **(18)**

Sol. $$A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}3\\3\\2\end{bmatrix} \text{ and } A\begin{bmatrix}3\\3\\2\end{bmatrix}=\begin{bmatrix}-3\\19\\-24\end{bmatrix}$$

det. (adj.(2 adj (A+I)) = $|2\text{ adj }(A+I)|^2 = 64\,|\text{adj }(A+I)|^2$

$= 64\,|A+I|^4$

$$\text{Let } A=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}\Rightarrow\begin{Bmatrix}-a=3\\-b+c=2\\3a+2b=-3\end{Bmatrix}$$

$$\Rightarrow\begin{cases}a=-3\\b=3\\c=5\end{cases}$$

$$\therefore\ |A+I| = =\begin{vmatrix}1&-3&3\\3&1&5\\-3&-5&1\end{vmatrix}=44$$

23. If $\int(\sin x)^{\frac{-11}{2}}(\cos x)^{\frac{-5}{2}}dx=$

$$-\frac{p_1}{q_1}(\cot x)^{\frac{9}{2}}-\frac{p_2}{q_2}(\cot x)^{\frac{5}{2}}-\frac{p_3}{q_3}(\cot x)^{\frac{1}{2}}+\frac{p_4}{q_4}(\cot x)^{\frac{-3}{2}}+C,$$

where p_i and q_i are positive integers with $\gcd(p_i, q_i) = 1$ for i = 1, 2, 3, 4 and C is the constant of integration, then $\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}$ is equal to ________.

Ans. **(16)**

Sol. $\int (\tan x)^{-11/2} .\sec^8 x\,dx$

$= \int (\tan x)^{-11/2} \left(1+\tan^2 x\right) 3\sec^2 x dx$

Put tan x = t

$\Rightarrow \int t^{-11/2}\left(1+t^2\right)^3 dx = \int t^{-11/2}\left(1+t^6+3t^2+3(4)\right)dt$

$= \int \left(t^{-11/2}+t^{1/2}+3.t^{-7/2}+3.t^{-3/2}\right)dt$

$= -\frac{2}{9}(\cot x)^{9/2} - \frac{6}{5}(\cot)^{5/2} - 6(\cot x)^{1/2} + \frac{2}{3}(\cot x)^{-3/2} + C$

$\Rightarrow p_1 = 2, p_2 = 6, p_3 = 6, p_4 = 2$

& $q_1 = 9, q_2 = 5, q_3 = 1, q_4 = 3$

$\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4} = \frac{15.2.6.6.2}{9.5.1.3} = 16$

24. If $\frac{\cos^2 48° - \sin^2 12°}{\sin^2 24° - \sin^2 6°} = \frac{\alpha + \beta\sqrt{5}}{2}$, where α, $\beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to ________.

Ans. **(4)**

Sol. Use $\sin(A + B) \sin (A + B) = \sin^2A - \sin^2B$

$\cos (A + B) \cos (A - B) = \cos^2A - \sin^2B$

$$\frac{\cos 60° \cos 36°}{\sin 30° \sin 18°} = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2}{4}$$

$$= \frac{3+\sqrt{5}}{2}$$

$\alpha = 3 ; \beta = 1$

So, $(\alpha + \beta) = 4$

25. Let ABC be a triangle. Consider four points p_1, p_2, p_3, p_4 on the side AB, five points p_5, p_6, p_7, p_8, p_9 on the side BC and four points p_{10}, p_{11}, p_{12}, p_{13} on the side AC. None of these points is a vertex of the triangle ABC. Then the total number of pentagons, that can be formed by taking all the vertices from the points p_1, p_2, ... p_{13}, is ________.

Ans. **(660)**

Sol. **Case 1 :**

2 from AB, 2 from BC, 1 from AC

$\binom{4}{2}.\binom{5}{2}.\binom{4}{1} = 6 \cdot 10 \cdot 4 = 240$

Case 2 :

2 from AB, 1 from BC, 2 from AC

$\binom{4}{2}.\binom{5}{1}.\binom{4}{2} = 6 \cdot 5 \cdot 6 = 180$

Case 3 :

1 from AB, 2 from BC, 2 from AC

$\binom{4}{1}.\binom{5}{2}.\binom{4}{2} = 4 \cdot 10 \cdot 6 = 240$